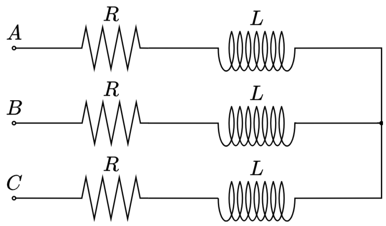
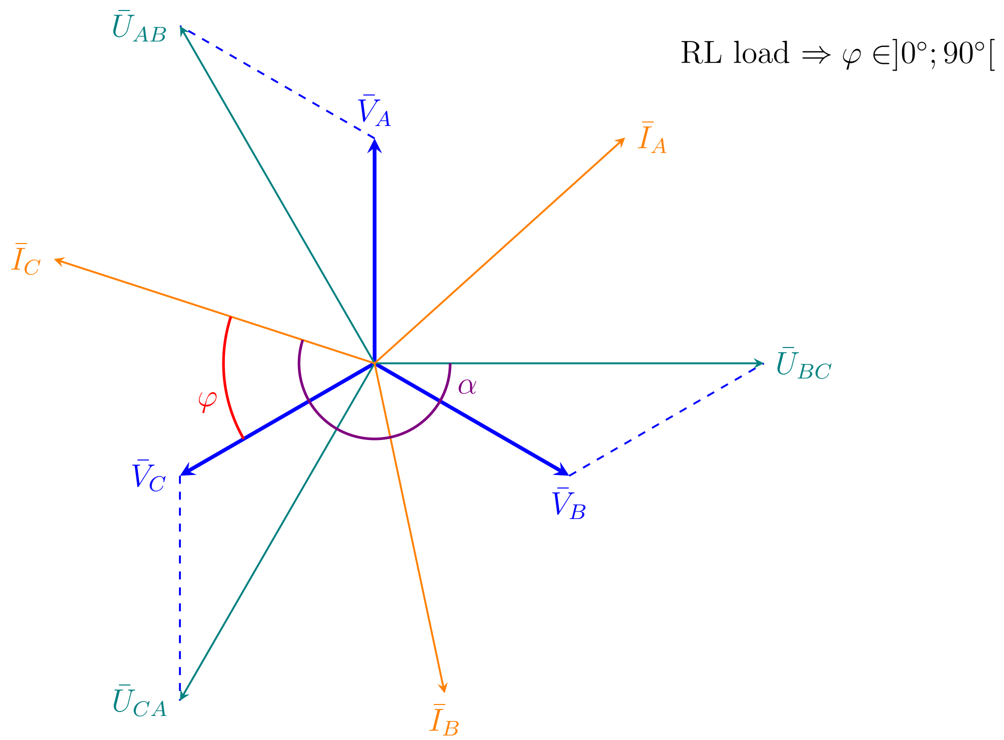
The answers to the june 2023 examination are found below. Please, note that, for questions 3 (Synchronous machine) and 5 (DC machine), rather detailed and elaborated answers are given in this correction for didactical purpose, but it was not necessary to give such detailed answers at the examination to have full mark. The important expected keywords are written in bold face.

Question 1



The above figure shows a 3-phase load connected in A,B,C to an equilibrated 3-phase power supply working at 50 Hz.

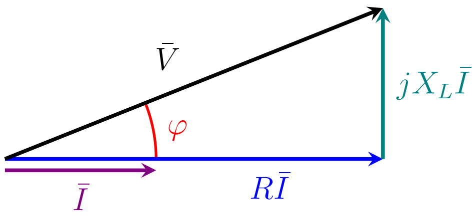
1. Complete the phasor diagram below by drawing the line voltages $\bar{U}_{AB}$, $\bar{U}_{BC}$ and $\bar{U}_{CA}$, and the line currents $\bar{I}_A$, $\bar{I}_B$ and $\bar{I}_C$. Make sure to respect ratios and angles.



2. The angle $\alpha = \angle\bar{U}_{BC} - \angle\bar{I}_C = 192°$ has been measured with an oscilloscope. Deduce the value of the phase shift φ between phase currents and corresponding phase voltages.

$$\alpha = \varphi + 120° + 30°$$
$$\varphi = 42°$$

3. With a DC test, R has been measured to be $20\,\Omega$. Determine the value of L.



$$tan(\varphi) = \frac{X_L}{R}$$
$$X_L = tan(\varphi)R = 2\pi f L$$
$$L = \frac{tan(\varphi)R}{2\pi f} = 57.3\,mH$$

4. One now wants to connect a capacitor in parallel with each branch of the load in order to compensate the reactive power of the load, i.e., achieve a zero total reactive power. What should be the reactance X_C of these capacitors ?

Method 1

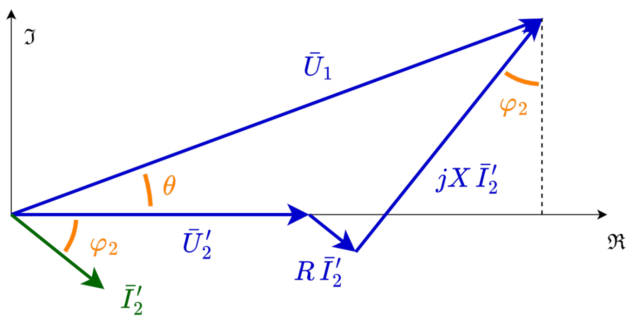
$$Q_L + Q_C = 0$$
$$X_L\,I_L^2 - X_C\,I_C^2 = 0$$
$$X_L\left(\frac{V}{\sqrt{R^2+X_L^2}}\right)^2 - X_C\left(\frac{V}{X_C}\right)^2 = 0$$
$$X_C = \frac{R^2+X_L^2}{X_L}$$

Method 2

$$\mathbb{I}\{Z_{tot}\} = 0$$
$$\mathbb{I}\left\{\left(\frac{1}{R+j\,X_L} + \frac{j}{X_C}\right)^{-1}\right\} = 0$$
$$\mathbb{I}\left\{\frac{R-j\,X_L}{R^2+X_L^2} + \frac{j}{X_C}\right\} = 0$$
$$X_C = \frac{R^2+X_L^2}{X_L}$$

Question 2

Transformers are practically pictured by electrical engineers by means of the phasor diagram corresponding to the simplified equivalent circuit. Consider the transformer described by the following phasor diagram :



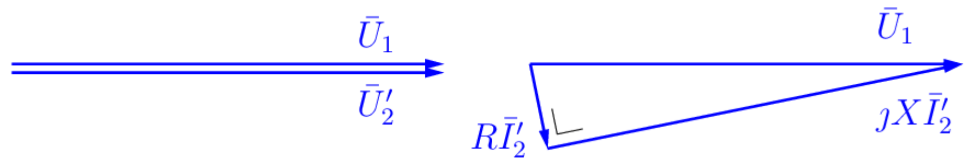
1. What is the detailed physical meaning of R and X, in terms of the resistances and inductances of the primary and secondary windings ?

$$R = R_1 + R_2' \quad , \quad X = X_1 + X_2'$$

The resistance R is the sum of the resistance R_1 of the primary windings plus the resistance $R_2' = (\frac{n_1}{n_2})^2 R_2$ of the secondary winding *as seen from the primary.*

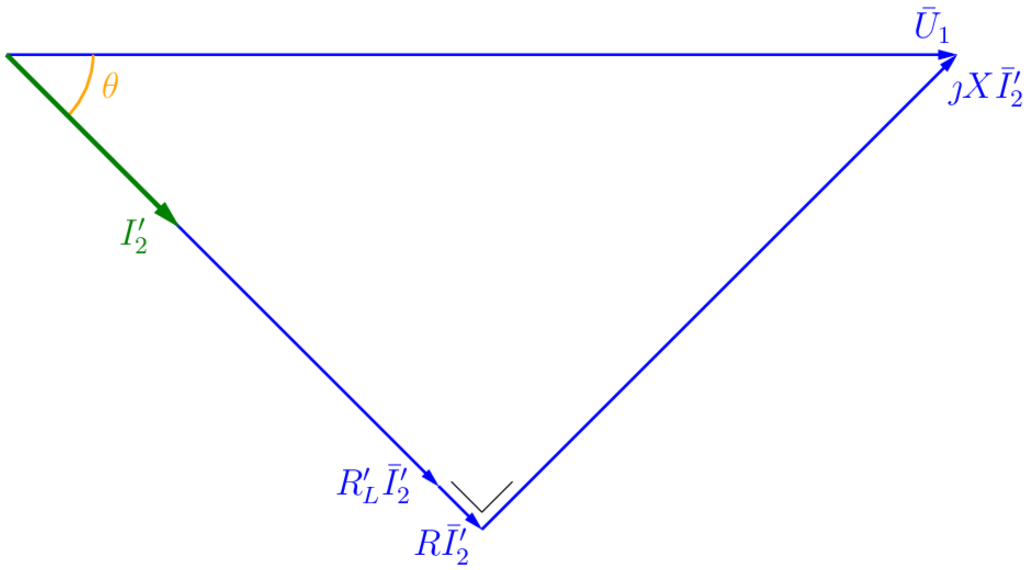
The reactance X is the sum of the reactance X_1 of the primary windings plus the reactance $X_2' = (\frac{n_1}{n_2})^2 X_2$ of the secondary winding *as seen from the primary.*

2. Draw the phasor diagram corresponding to the no-load experiment and the short-circuit experiment. Carefully label all phasors. Indicate also the current phasors. Do not neglect R.



No-load (left) : $\bar{I}_2' = 0$
Short-circuit (right): $\bar{U}_2' = 0$

3. Assume the phasor diagram above corresponds to a single-phase transformer operating at 50 Hz with $U_1 = 220\,\text{V}$, $R = 1\,\Omega$, $X = 11\,\Omega$, and a transformation ratio $n_1/n_2 = n = 10$. The secondary of the transformer is connected to a resistance $R_L = 0.1\,\Omega$. What is the value R_L' of the load reasistance seen from the primary ? Draw the phasor diagram corresponding to this situation with the scale 20 V/cm. Place the phasor $\bar{U}_1$ horizontally. Do not neglect R.



$R_L' = n^2 R_L = 10^2\, 0.1\,\Omega = 10\,\Omega$

Important remark: R and X are composite quantities containing a (non-primed) quantity from the primary and a (primed) quantity from the secondary as seen from the primary. Conventionally, these quantities R and X are not represented with a prime in the simplified phasor diagram of the transfo.

The resistive load, on the other hand, is a true secondary quantity, so it must be represented in the phasor diagram and in the calculations by its primed version $R_L' = n^2 R_L$ as seen from the primary.

4. What are the values of φ_2, θ, I_2' and I_2? Assuming X_μ and R_{H+F} are very large, estimate the current I_1 in the primary winding. What is the active power P_1 delivered at the primary of the transformer, and the active power P_2 received at the secondary?

$\varphi_2 = 0$ because the load is a resistance.
$\theta = 45°$ as seen from diagram
$X I_2' = 220\,\text{V}/\sqrt{2} = 155.56\,\text{V}$ from diagram
$I_2' = 155.56\,\text{V}/11\,\Omega = 14.14\,\text{A}$
$U_2' = R_L'\, I_2' = 141.4\,\text{V}$
$I_2 = n\, I_2' = 141.4\,\text{A}$
$U_2 = U_2'/n = 14.14\,\text{V}$
If $X_\mu, R_{H+F} >> $ one has $\bar{I}_1 \approx \bar{I}_2'$ and therefore $I_1 = 14.14\,\text{A}$ and $\varphi_1 = 45°$

$$P_1 = U_1 I_1 \cos\varphi_1 = 220\,\text{V}\,14.14\,\text{A}\sqrt{2}/2 = 2200\,\text{W}$$
$$= (R_L' + R)(I_2')^2 = (10\,\Omega + 1\,\Omega)(14.14\,\text{A})^2 = 2200\,\text{W}$$

$$P_2 = U_2' I_2' \cos\varphi_2 = 141.4\,\text{V}\ 14.14\,\text{A}\ 1 = 2000\,\text{W}$$
$$= R_L'(I_2')^2 = 10\,\Omega\ (14.14\,\text{A})^2 = 2000\,\text{W}$$
$$= U_2 I_2 \cos\varphi_2 = 14.14\,\text{V}\ 141.4\,\text{A}\ 1 = 2000\,\text{W}$$
$$= R_L(I_2)^2 = 0.1\,\Omega\ (141.4\,\text{A})^2 = 2000\,\text{W}$$

Question 3

1. Explain the difference between the Potier diagram and the Behn-Eschenburg diagram for synchronous alternators.

The Potier diagram is a theoretical phasor diagram that describes the behavior of a synchronous machine (e.g., an alternator). It involves the e.m.f. $E_r(I_r)$ created in the stator windings by the magnetic flux **produced by both the excitation current I_e and the reaction current I**, since one has $\bar{I}_r = \bar{I}_e + \frac{\delta}{\gamma}\bar{I}$. The reactance X_f that appears in Potier diagram represents the **leakage fluxes of the stator windings** of the machine.

In the Behn-Eschenburg diagram, the e.m.f. E_r (which cannot be measured) is replaced by the no-load e.m.f. $E_v(I_e)$, which can be easily measured by means of a no-load experiment. In consequence, the reactance X_f is replaced by a fictitious reactance X_S, called **synchronous reactance**, that accounts not only for the leakage fluxes of the stator windings of the machine **but also for the effect of the reaction I.**

The Behn-Eschenburg diagram is equivalent with the Potier diagram if the condition

$$\frac{E_r}{I_r} = \frac{E_v}{I_e} = \text{constant}$$

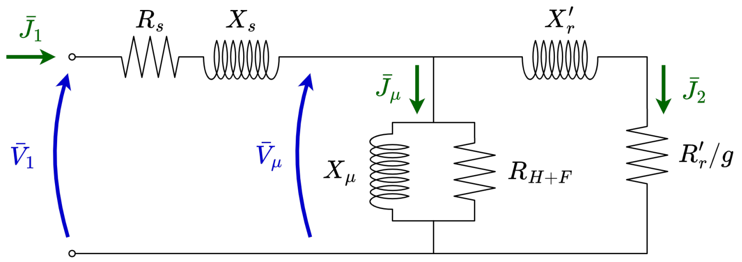
is fulfilled, which is the case whenever the machine is **not saturated**, otherwise, it is an approximation.

2. The alternator is connected to the power grid and delivers a fixed active power P. If the excitation current I_e of the alternator is increased, does the current I delivered to the grid by the stator windings increase or decrease? On what does the answer to this question depend?

The behavior of an alternator connected to the grid and delivering a constant active power P is described by a **Mordey V-curve**. The minimum of this curve corresponds to the situation where $\varphi = 0$ and the alternator exchanges no reactive power with the grid ($Q = 0$). Let $I_{e,Q=0}$ be the excitation current corresponding to that situation.

Now, according to the Mordey curve, one has:
If $I_e < I_{e,Q=0}$, an increase of the excitation current I_e yields a decrease of the stator current I.
If $I_e > I_{e,Q=0}$, an increase of the excitation current I_e yields an increase of the stator current I.

Question 4



A three-phase asynchronous motor is studied, and the equivalent electric circuit of one of its phases is depicted above. The values $R_s = 0.7\,\Omega$ and $X_s = 4\,\Omega$ have been determined from prior measurements. Two additional tests are conducted to calculate the values of X_μ, R_{H+F}, X'_r and R'_r.

- In the first test, the rotor is rotating at synchronous speed. An input RMS phase current J_1 of 479 mA and an input RMS phase voltage V_1 of 230 V are recorded. Additionally, the input three-phase active power $P_{1,3\varphi}$ is measured to be 264.5 W.
- In the second test, the shaft is locked to prevent its rotation. An input RMS phase current J_1 of 3 A, an input RMS phase voltage V_1 of 28.7 V, and an input three-phase active power $P_{1,3\varphi}$ of 86.4, W are measured.

1. What assumptions are usually made to simplify the calculation of X_μ, R_{H+F}, X'_r and R'_r from the experimental values obtained with the above-mentioned tests?

$$R_s,\ R'_r \ll R_{H+F} \quad \& \quad X_s,\ X'_r \ll X_\mu$$

2. Determine the values of X_μ and R_{H+F}. Use the assumptions made at point 1.

First test
($\dot{\theta} = \dot{\theta}_s \rightarrow g = 0 \rightarrow J_2 = 0\,A$, voltage drop across
R_s and X_s negligible since $R_s \ll R_{H+F}$ and $X_s \ll X_\mu$)

$$|S| = V_1\,J_1$$
$$P = P_{1,3\varphi}/3$$
$$Q = \sqrt{|S|^2 - P^2}$$
$$P = V_1^2/R_{H+F} \rightarrow R_{H+F} = V_1^2/P = 600\,\Omega$$
$$Q = V_1^2/X_\mu \rightarrow X_\mu = V_1^2/Q = 800.8\,\Omega$$

3. Determine the values of X'_r and R'_r. Use the assumptions made at point 1.

Second test
($\dot{\theta} = 0 \rightarrow g = 1$, current in R_{H+F} and X_μ
negligible since $R'_r \ll R_{H+F}$ and $X'_r \ll X_\mu$)

$$|S| = V_1\,J_1$$
$$P = P_{1,3\varphi}/3$$
$$Q = \sqrt{|S|^2 - P^2}$$
$$P = (R_s + R'_r)\,J_1^2 \rightarrow R'_r = P/J_1^2 - R_s = 2.5\,\Omega$$
$$Q = (X_s + X'_r)\,J_1^2 \rightarrow X'_r = Q/J_1^2 - X_s = 5\,\Omega$$

(Assumptions of point 1 ok)

For the next points, assume $R_s = 0.7\,\Omega$, $X_s = 4\,\Omega$, $R_{H+F} = 600\,\Omega$, $X_\mu = 800\,\Omega$, $X'_r = 5\,\Omega$ and $R'_r = 2.5\Omega$.

The asynchronous motor is now driving a mechanical load. The motor draws an input RMS phase current J_1 of 4.842 A and an input RMS phase voltage V_1 of 230 V. Additionally, it is observed that $\bar{J}_1$ is lagging $\bar{V}_1$ by a phase angle $\varphi = 13.207°$.

4. What are the values of the active power P_{st-rot} and of the RMS current J_2? Do not neglect any component of the equivalent circuit.

Method 1

$$P_{in} = V_1\, J_1\, cos(\varphi)$$
$$Q_{in} = V_1\, J_1\, sin(\varphi)$$
$$P_\mu = P_{in} - R_s\, J_1^2$$
$$Q_\mu = Q_{in} - X_s\, J_1^2$$
$$|S_\mu| = \sqrt{P_\mu^2 + Q_\mu^2}$$
$$V_\mu = |S_\mu|/J_1$$
$$P_{st-rot} = P_\mu - V_\mu^2/R_{H+F} = 984.7\,W$$
$$Q_{st-rot} = Q_\mu - V_\mu^2/X_\mu$$
$$|S_{st-rot}| = \sqrt{P_{st-rot}^2 + Q_{st-rot}^2}$$
$$J_2 = |S_{st-rot}|/V_\mu = 4.439\,A$$

Method 2

$$\bar{V}_1 = 230\angle 0°\ [V]$$
$$\bar{J}_1 = 4.482\angle -\varphi\ [A]$$
$$\bar{V}_\mu = \bar{V}_1 - (R_s + j\,X_s)\,\bar{J}_1$$
$$\bar{J}_\mu = \bar{V}_\mu\,(1/R_{H+F} + 1/(j\,X_\mu))$$
$$\bar{J}_2 = \bar{J}_1 - \bar{J}_\mu$$
$$S_{st-rot} = \bar{V}_\mu\,\bar{J}_2^* = P_{st-rot} + j\,Q_{st-rot}$$
$$J_2 = |\bar{J}_2|$$

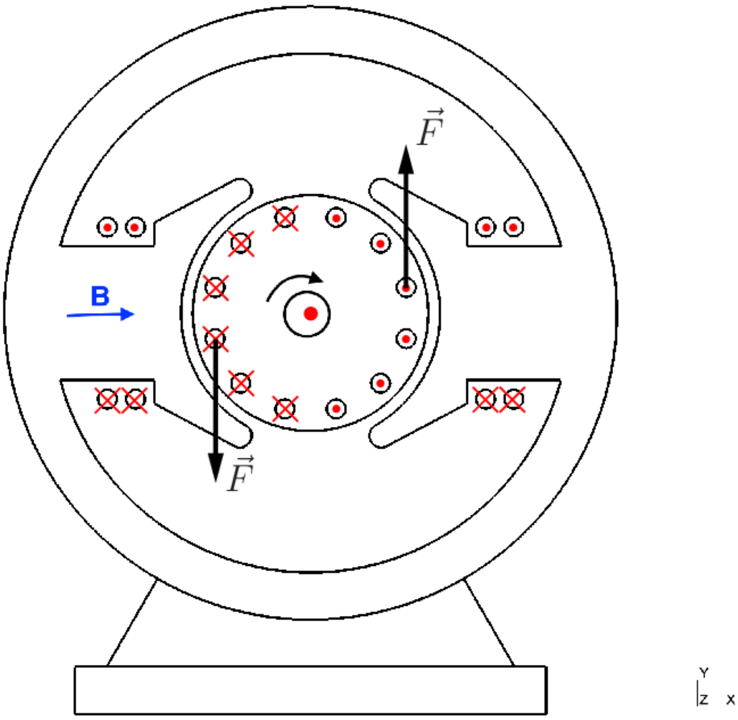
5. Find the value of the slip g.

$$P_{st-rot} = \frac{R'_r}{g}\,J_2^2 \rightarrow g = \frac{R'_r\,J_2^2}{P_{st-rot}} = 0.05$$

Question 5

For this question, use a drawing pencil (crayon à dessin), so as to be able to erase and correct neatly.

The picture below represents a DC *generator* in operation. The direction of the inductor **B** field is given by the vector on the left stator pole, and the direction of rotation of the machine is indicated by the curved arrow near the shaft.



1. What is the direction of the torque? On basis of those pieces of information, indicate the orientation of the currents in all excitation (stator) conductors and armature (rotor) conductors, using the symbols ⊗ for a current towards the page and ⊙ for a current outside the page (parallel to the Z axis). Explain your reasoning and give the physical laws you have used. Assume the brushes are centered on the neutral axis (i.e., they are not shifted).

Stator currents are oriented with the **right-hand rule**, knowing the directions of the inductor $\vec{B}$ field indicated on the figure (See figure above).

In a generator, the torque is **opposed to the direction of rotation** of the rotor (opposing torque). Considering a rotor conductor in the left half of the machine, the force vector $\vec{F}$ must be oriented to the bottom ($-Y$ direction) to generate an opposing torque. The direction of the current in that conductor is then determined, **thanks to $d\vec{F} = Id\vec{L} \times \vec{B}$, to be oriented in $-Z$ direction (piercing the page)**. Same procedure for a conductor of the right half of the machine. As the brushes are not shifted, all conductors of the left half carry a current with same orientation ($-Z$), and all conductors of the right half carry a current along the $+Z$ direction.

Alternatively, one can use the fact that the e.m.f. E is in the direction of the current I in a generator. Knowing the direction of rotation $\dot{\theta}$, one knows the velocity vector $\vec{v}$ of the rotor conductors, hence the direction of the electric field $\vec{e} = \vec{v} \times \vec{B}$, and the direction of the current density $\vec{j} = \sigma\vec{e}$, which also allows to orient the current in all rotor conductors.

2. In the same situation as above, sketch in the figures below the field lines due to the armature currents only (in the left picture), and the resulting field lines due to both the armature and the excitation currents (in the right picture). A couple of field lines per figure is enough.

   Refer for this to slide 4 of the courses on DC machines.

3. With this picture, explain the concept of armature reaction in DC machines. What is the property of magnetic materials (steel, iron) that causes this phenomenon? Give the meaning of the different terms of the equation

$$E = E_v - \psi(I_a).$$

   How does $\psi(I_a)$ depend on the rotation speed $\dot{\theta}$?

   The armature reaction in a DC generator tends to weaken the $\vec{B}$ field at the pole entries and to reinforce it at the pole exits (it is the opposite in a DC motor).

   This $\vec{B}$ field distortion does not affect the e.m.f. E if the magnetic cores of the machine are not saturated, because local field increase and the local field decrease cancel out to deliver the same total flux embraced by the rotor conductors, Hence, the e.m.f. E, is equal to the the no-load e.m.f. E_v in the absence of armature reaction ($I_a = 0$).

   If the magnetic cores are saturated (it is **saturation** that plays a role here, not hysteresis), this compensation no longer occurs and **the flux embraced by the rotor conductors is reduced by an amount** $\Delta\Phi(I_a)$ [Wb], $\Delta\Phi(I_a = 0) = 0$.

   The armature reaction

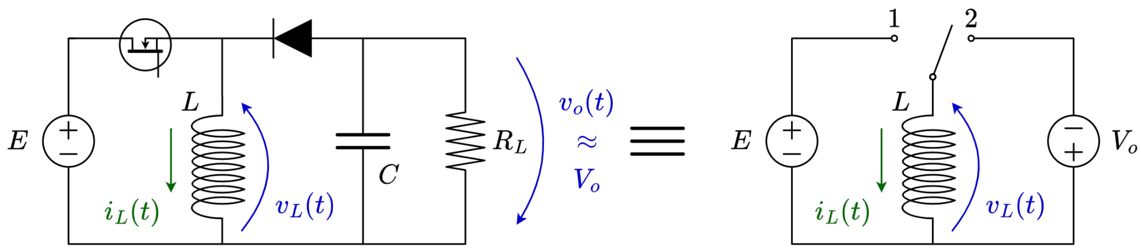
$$\psi(I_a) = k_E \dot{\theta} \Delta\Phi(I_a) \quad [V]$$

   is the **correspondig reduction of the e.m.f.** E, so that one has

$$E = E_V - \psi(I_a).$$

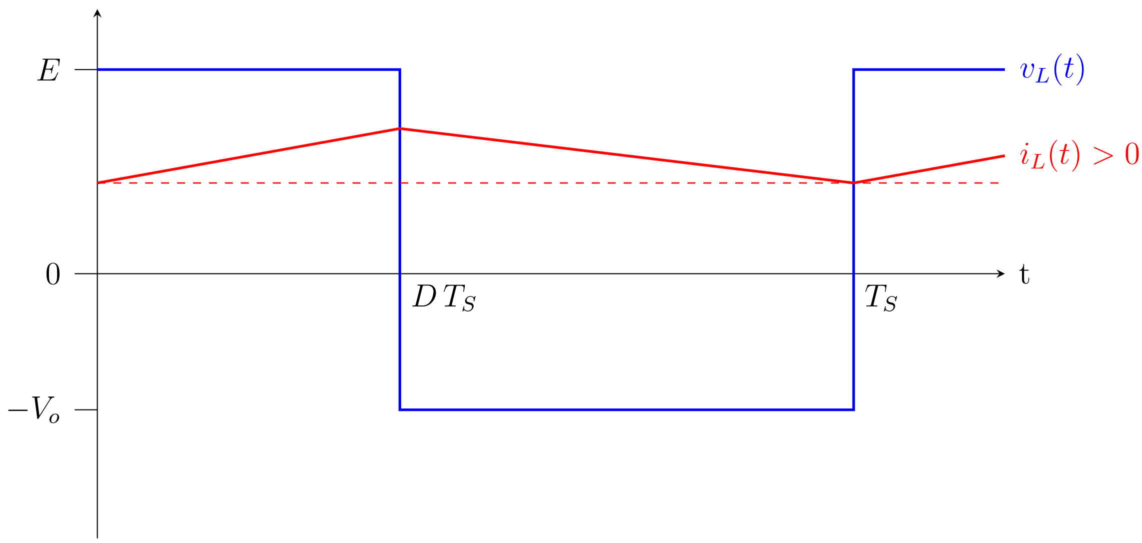
   The armatutre reaction $\psi(I_a)$ is **proportional to the rotation speed** $\dot{\theta}$.

Question 6



The above figures present a buck-boost DC-DC converter.

1. Draw the waveform of the voltage across the inductance ($v_L(t)$) in steady-state condition, and deduce from it the current waveform in the inductance ($i_L(t)$). Draw it on the same diagram using another color.



2. Assuming again the steady-state condition, express the ratio $\frac{V_o}{E}$ in terms of the duty cycle D.

Volt-second balance: $\langle v_L(t) \rangle = 0$

$$\int_0^{T_s} v_L(t)\, dt = 0$$

$$D\,E + (1 - D)\,(-V_o) = 0$$

$$\frac{V_o}{E} = \frac{D}{1-D}$$