Electromagnetic Energy Conversion ELEC0431

Exercise session 4: Three-phase transformers

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In this class...

- Organization point: test and laboratories
- Exercise 9

Organization point: test and laboratories

Test on phasors in the sinusoidal steady state and three-phase systems:

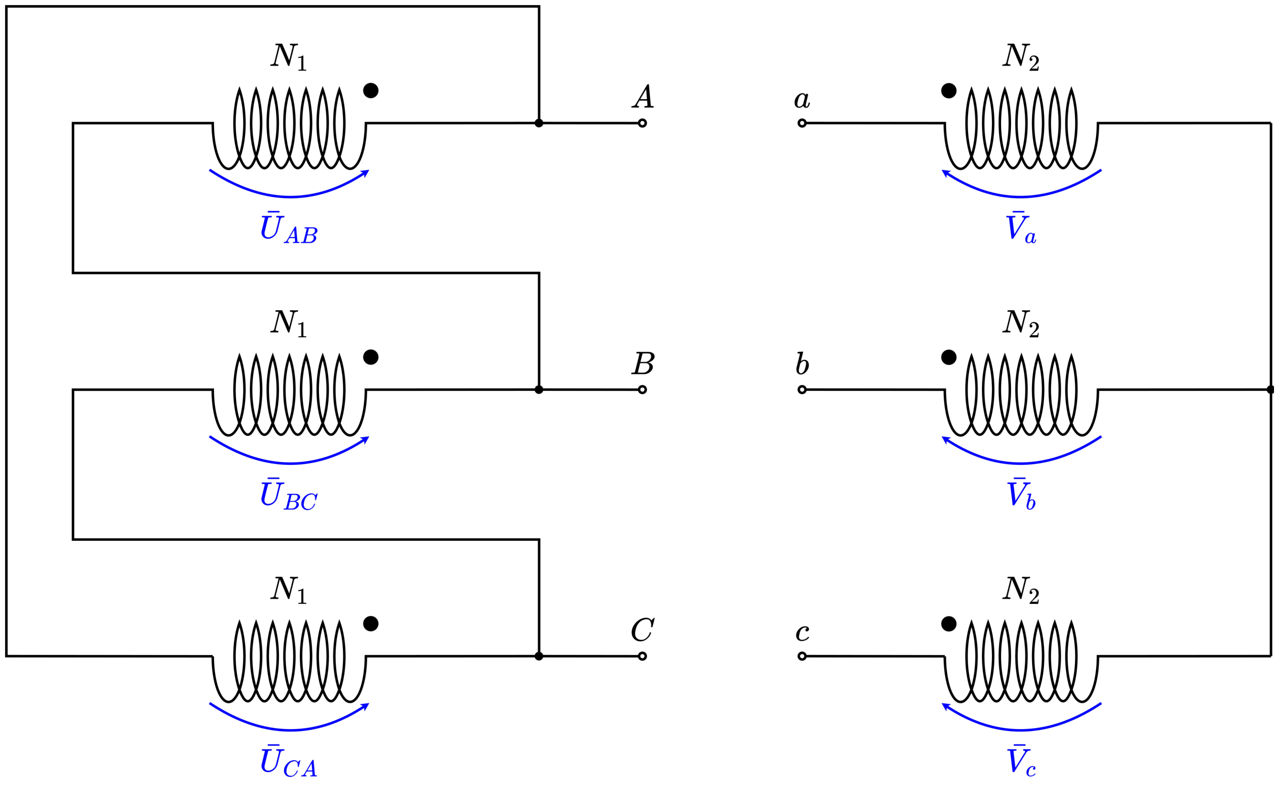
- The test is scheduled for next week (Friday 8th of March) at 9:00 am.
- It takes place in the usual classroom (B37 auditorium 02).
- The test focuses on the material seen during the first and second exercise sessions. You can practice by solving the related homeworks.
- To take the test, take with you your calculator, ruler and protractor.
- This is a quick test of 30 minutes. It will be followed by its correction and a normal exercise session.

Laboratory schedule:

- The schedule for the laboratory sessions is now available on the course webpage.
- Make sure to read it and to update your own calendars accordingly!

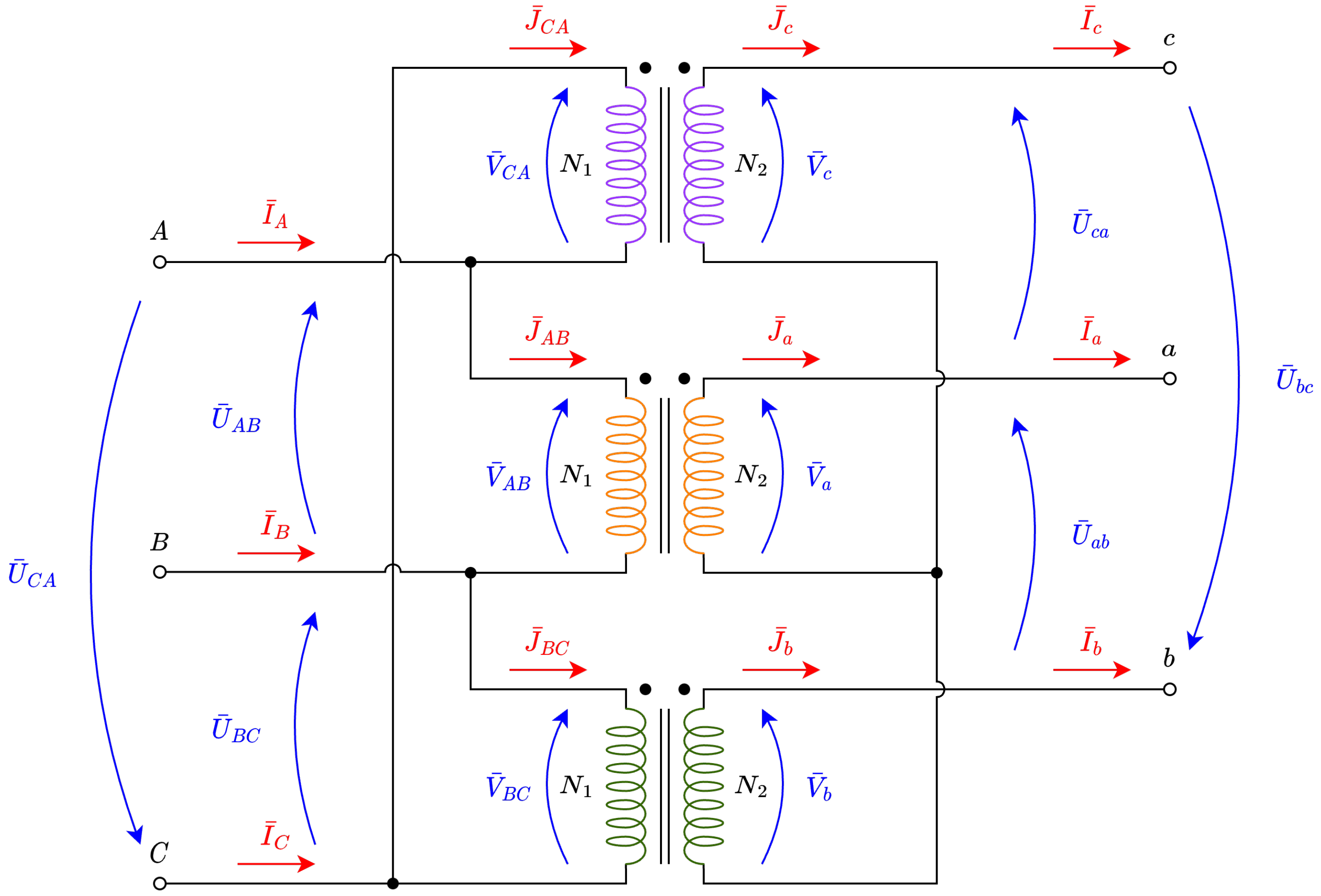
Exercise 9

Three-phase power transformers are commonly used to adapt power line voltages and to provide some galvanic insulation between two parts of an electrical grid. The three-phase transformer, described by the normalized scheme hereunder, is connected to a balanced three-phase network of line voltages $\overline{U}_{AB}$, $\overline{U}_{BC}$ and $\overline{U}_{CA}$ of RMS voltage U_1 on the primary side, whereas on the secondary side, a three-phase balanced system of line voltages $\overline{U}_{ab}$, $\overline{U}_{bc}$ and $\overline{U}_{ca}$ of RMS voltage U_2 is obtained. The line current intensities in the primary and secondary windings are respectively denoted I_1 and I_2.



Exercise 9

Another way to draw the circuit:



- Each winding at the primary is linked to a winding at the secondary.
- ➔ When the system is balanced, one can solve it by considering only one phase.
- We define:
  - The column ratio n_c. That is the ratio between primary and secondary **phase voltages/currents**:

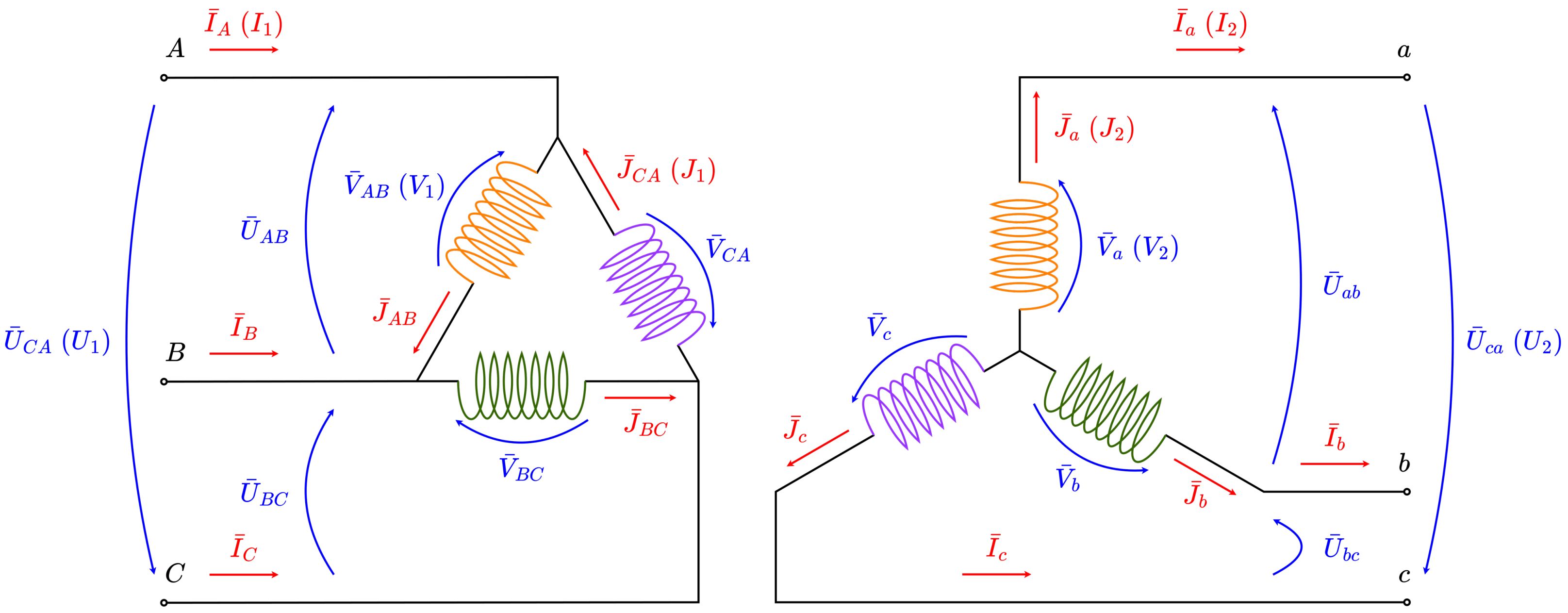
$$n_c = \frac{V_2}{V_1} = \frac{J_1}{J_2} \quad \text{or} \quad n_c = \frac{V_1}{V_2} = \frac{J_2}{J_1}.$$

  - The transformer ratio n. That is the ratio between primary and secondary **line voltages/currents**:

$$n = \frac{U_2}{U_1} = \frac{I_1}{I_2} \quad \text{or} \quad n = \frac{U_1}{U_2} = \frac{I_2}{I_1}.$$

Exercise 9

Another way to draw the circuit:

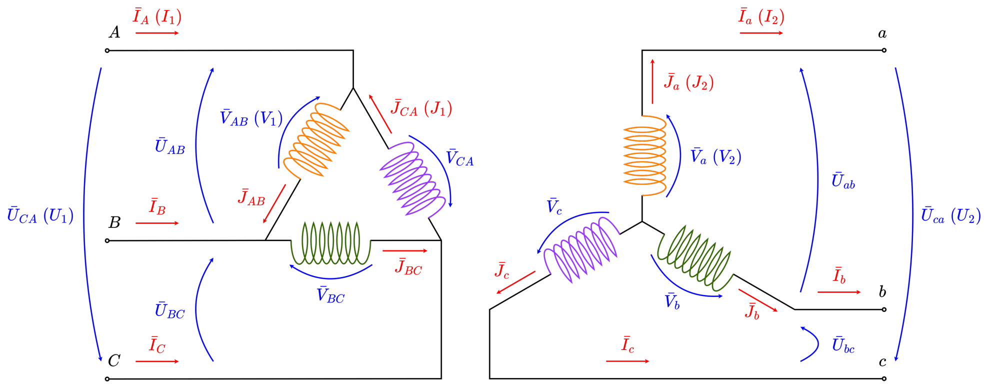


One can pass from the column ratio n_c to the transformer ratio n (and inversely) if the primary and secondary configurations (delta or star configurations) are known.

Here, with a primary delta configuration and secondary star configuration:

$$n_c = \frac{n_2}{n_1} = \frac{V_2}{V_1} = \frac{U_2/\sqrt{3}}{U_1} = \frac{n}{\sqrt{3}}$$

Exercise 9



Transformer characteristics:

- Apparent nominal power $|S_n| = 250$ kVA
- Line primary-winding nominal RMS voltages $U_{1n} = 5.2$ kV
- Nominal frequency $f_n = 50$ Hz
- Ferromagnetic losses are neglectable

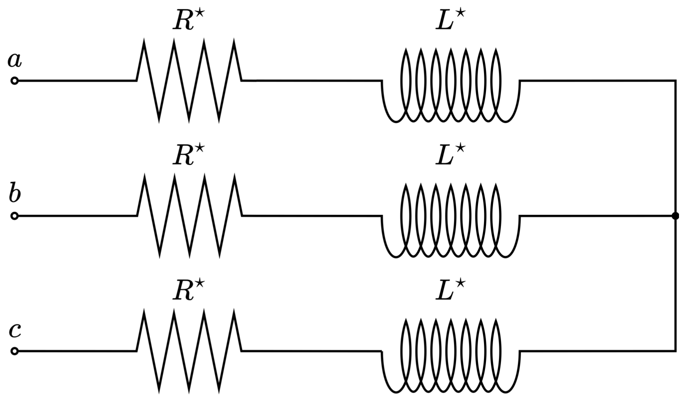
Two tests have been performed to characterize the transformer:

- With open secondary windings, $U_{2o} = 400$ V & $U_{1o} = U_{1n}$.
- With short-circuited secondary windings, $U_{1s} = 600$ V, $P_{3\varphi} = 7.35$ kW & $I_{2s} = 350$ A.

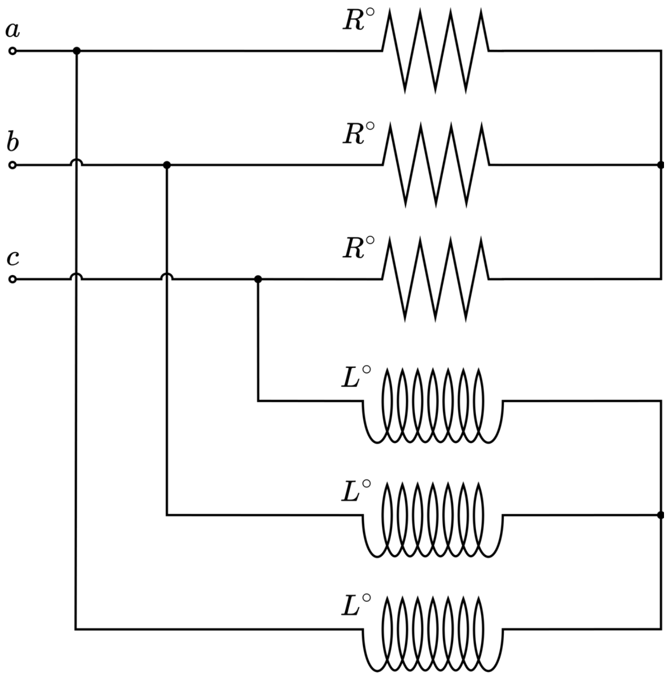
1. Calculate the transformer ratio n so that it is greater than 1.
2. For the first test condition (open circuit), draw a phasor diagram including the primary line voltages, the secondary phase voltages and the secondary line voltages.
3. Express and compute the column ratio n_c according to the transformer ratio n.
4. The transformer is composed of 3 cores of section $A_c = 5\,dm^2$ and the magnetic field amplitude is $B_m = 1.2$ T. Compute the number of turns of each primary winding and the number of turns of each secondary winding.
5. Using a single-phase equivalent model (impedances gathered and moved to the secondary windings), provide the Thevenin's model seen from a secondary winding and calculate the resistance R' and the reactance X'.

Exercise 9

The nominal regime is now considered by applying the line nominal voltage U_{1n} at the primary windings and connecting a three-phase balanced load on the secondary side (detailed here on right). Each branch is composed of a resistor of value $R^\star = 554\,m\Omega$ in series with a coil of value $L^\star = 3.05\,mH$.



6. Calculate the power factor $\cos(\varphi_2)$ of this load.
7. Draw the phasor diagram corresponding to the balanced single-phase equivalent model. Deduce the RMS values of the line current intensities I_2 and of the line voltage intensities U_2.
8. Compute the active power P_2 flowing from the transformer to the load.
9. Calculate the transformer efficiency η.
10. Another load is used, compute the value of the resistance R° and the inductance L° such that this load is equivalent to the one detailed in previously.



Homework 17

In Belgium, most of the railways are powered using DC 3 kV voltage. High speed train lines are however supplied with AC 25 kV 50 Hz (single-phase) voltage, requiring the use of high-power single-phase transformers. In this exercise, such a transformer is considered. A nominal RMS voltage of $U_{1n} = 25$ kV with nominal frequency $f = 50$ Hz is supplied to the primary winding with an apparent power $|S_n| = 5.6$ MVA.

To characterize the transformer two tests have been performed:

- Using open secondary winding with the nominal voltage applied to the primary, the transformer generates a voltage $U_{2o} = 1.36$ kV at the secondary winding, for a current drawn at the primary $I_{1o} = 1.25$ A, and a consumed active power $P_{1o} = 6.8$ kW.
- Using short-circuited secondary winding, the transformer consumes an active power $P_{1s} = 25$ kW, considering that a reduced voltage of 10.1 % of U_{1n} was applied to the primary winding to maintain the secondary winding current to its nominal value I_{2n}.

1. Calculate the transformer ratio n.
2. Determine the nominal RMS secondary current I_{2n} and primary current I_{1n}.
3. Compute the power factor $\cos \varphi_{1o}$ for the first test (open secondary winding) and deduce the phase shift φ_{1o} of the current at the primary winding with respect to the primary winding voltage.
4. Give the reactive power Q_{1o} for the first test (open secondary winding).

Homework 17 – Cont'd

5. Considering the model of a transformer with impedances moved and gathered at the primary, calculate the resistance R_{H+F} and the magnetizing inductance L_{μ}.
6. Compute the RMS current intensity I_{2s} in the secondary winding for the second test (shorted secondary winding), compute the primary winding voltage U_{1s}, and calculate the values of the resistance R and of the inductance L of the primary winding in the equivalent model.
7. Considering that L is chosen large enough to provide sufficient smoothing at the input of the single-phase rectifiers, compare the values of R and $X = 2\pi f L$ and propose a simplified version of the equivalent model of the transformer.

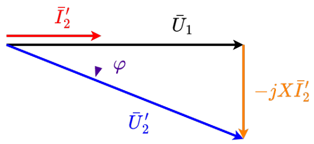
The nominal regime is now considered by applying the nominal voltage U_{1n} at the primary winding and connecting a load at the secondary winding, drawing an RMS current $I_2 = 4.097$ kA with a power factor $\cos\varphi_2$, the current being ahead on the voltage. The current $\bar{I}_2$ (or I'_2) in the secondary winding is aimed to be in phase with the voltage $\bar{U}_1$ of the primary winding.

8. Build the corresponding Fresnel diagram, clearly identifying the load voltage $\bar{U}'_2$.
9. Compute the phase shift φ_2 of the current $\bar{I}_2$ with respect to $\bar{U}_2$, and deduce the load power factor $\cos\varphi_2$.
10. Compute the RMS voltage value U_2 appearing at the secondary winding;

Homework 17 – Cont’d

11. Compute the reactive power Q_2 drawn by the load at the secondary winding, and the reactive power Q_1 at the primary winding.
12. Compute the active power P_2 drawn by the load at the secondary winding, and the active power P_1 at the primary winding.
13. Compute the transformer efficiency η.

Answers:

1. $n = 18.382$
2. $I_{2n} = 4117.647$ A, $I_{1n} = 224$ A
3. $\cos\varphi_{1o} = 0.2176$, $\varphi_{1o} = 77.432°$
4. $Q_{1o} = 30501.189$ var
5. $R_{H+F} = 91911.765\ \Omega$, $L_\mu = 65.225$ H
6. $I_{2s} = 4117.647$ A, $U_{1s} = 2525$ V, $R = 498.227\ m\Omega$, $L = 35.845$ mH
7. $R \ll X \rightarrow R$ can be neglected
8. 

9. $\varphi_2 = -5.733°$, $\cos\varphi_2 = 0.994998$
10. $U_2 = 1366.863$ V
11. $Q_2 = -559.4$ kvar, $Q_1 = 32.15$ kvar
12. $P_2 = 5572$ kW, $P_1 = 5604$ kW
13. $\eta = 99.43$ %