Electromagnetic Energy Conversion ELEC0431

Exercise session 3: Magnetic circuits and transformers

23 February 2024

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Reminder: Laboratories – Schedule and groups

To create the laboratory schedule, you are required to fill the doodle "https://cally.com/pmxdpjysb4mnkmzv":

- By group of **4 students**, select **AT LEAST 6** available time slots (you may be given random sessions if less than 6 time slots were selected).
- In the space provided for names, write the **STUDENT ID NUMBERS** of all **4 MEMBERS** of the group (for example: "s161514, s171856, s164442, s179088").
- In the space provided for emails, write the email of **ONE reference student** in the group (which will be contacted in case of issues with the schedule).
- A time slot can be selected by maximum six groups, **do not delay in completing this Doodle**.
- In addition to your selected time slots, your schedule could include a laboratory session on the 19/04, 26/04, 10/05 and 17/05 mornings (Friday mornings in place of the traditional classes).

IF AND ONLY IF it is impossible for you to create a group of four students meeting the requirements, send me an email (florent.purnode@uliege.be).

Make sure to complete the doodle by 23:59 on Friday, February 23rd,
(Students who would not have given their availabilities by this time will be given random time slots).

In this class...

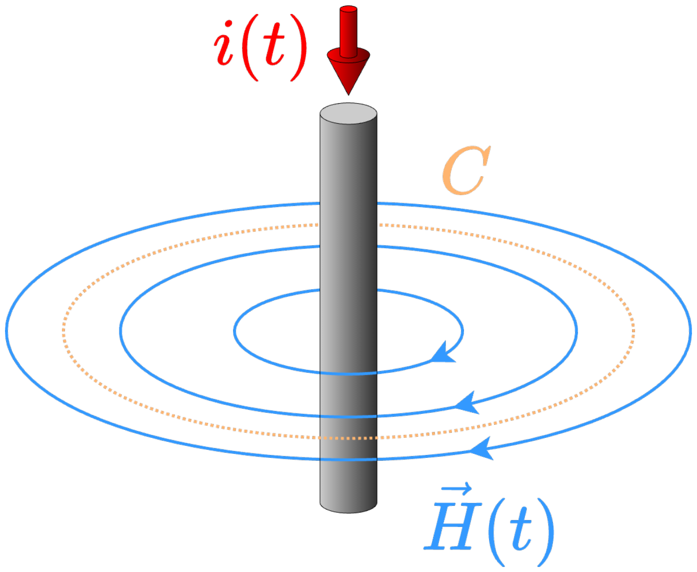
- Magnetic circuits
- Exercise 6
- Transformers
- Exercise 7

Magnetic circuits

Ampere’s law and magnetomotive force
Magnetic permeability and magnetic flux
Ferromagnetic materials
Reluctance and magnetic circuit
Exercise 6

Ampere's law and magnetomotive force

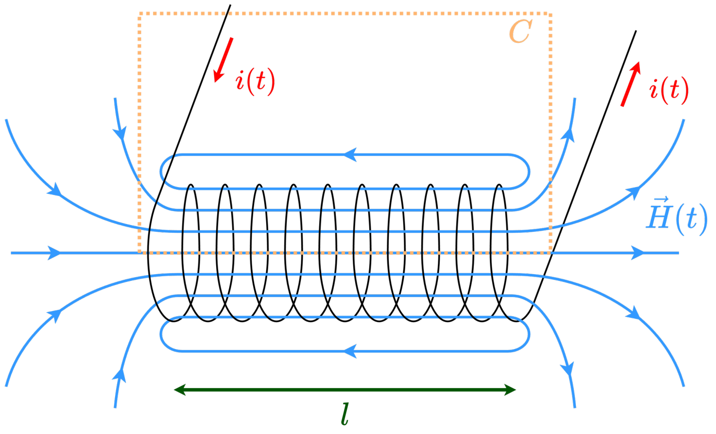
A current flowing into a wire generates a magnetic field H.



Ampere's law relates the magnetic field $\vec{H}(t)$ circulating around a closed loop C to the current $i(t)$ passing through that loop:

$$\oint_C \vec{H}(t) \cdot d\vec{l} = i(t)$$

A solenoid is a coil of wires.



Considering N turns, the magnetic field $\vec{H}(t)$ generated is N times the magnetic field for a single wire:

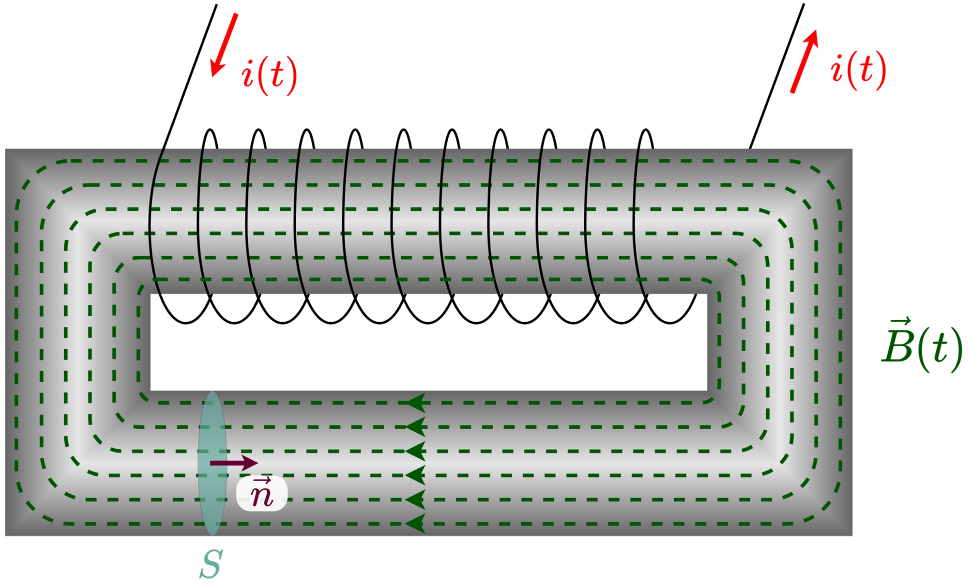
$$\oint_C \vec{H}(t) \cdot d\vec{l} = N\, i(t)$$

Inside of the coil, considering RMS values, it simplifies to:

$$H\, l = N\, I = \mathcal{F}$$

$\mathcal{F}$ is the **magnetomotive force**.

Magnetic permeability and magnetic flux



The magnetic permeability μ links Magnetic flux $\vec{H}$ and magnetic flux density $\vec{B}$:

$$\vec{B} = \mu\vec{H} = \mu_0\mu_r\vec{H}$$

- $\mu_0 = 4\pi 10^{-7} H/m$ is the permeability of vacuum
- μ_r is the relative permeability. Its value varies from one material to the other ($\mu_r = 1$ for air).

The magnetic flux $\phi(t)$ is the quantity of magnetic flux density $\vec{B}(t)$ crossing a surface S:

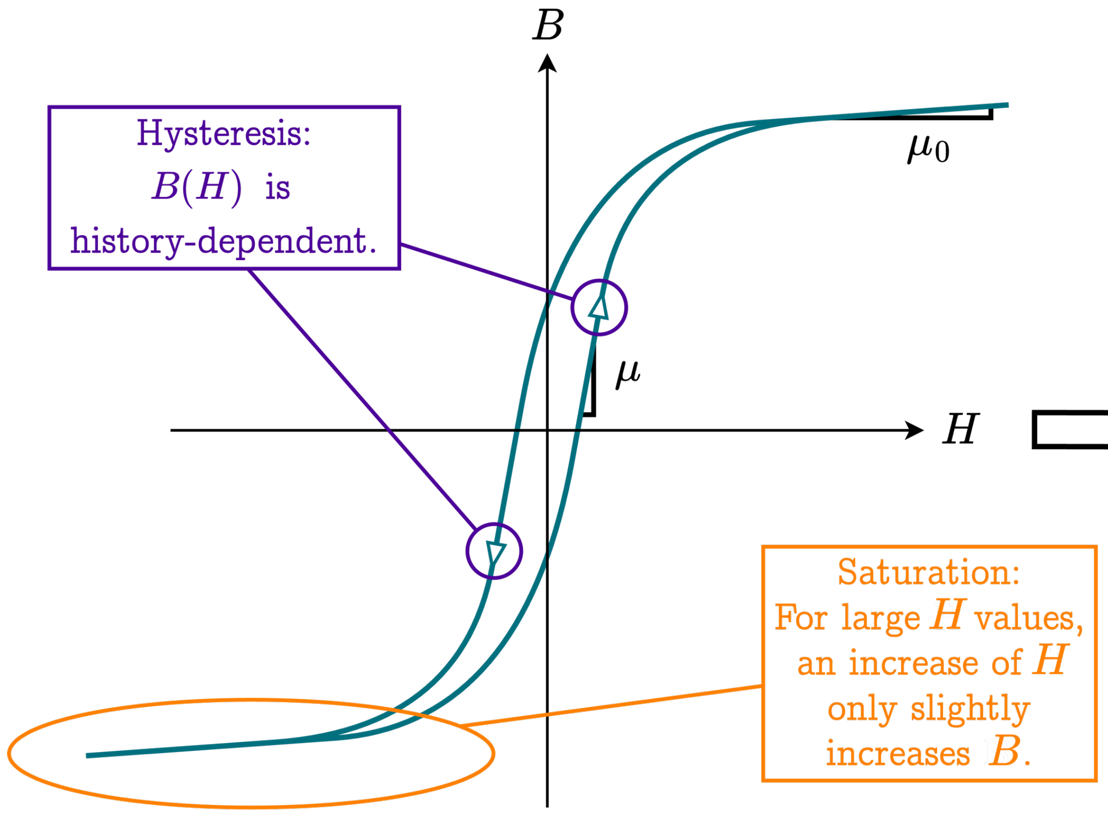
$$\phi(t) = \int_S \vec{B}(t) \cdot \vec{n}\ ds.$$

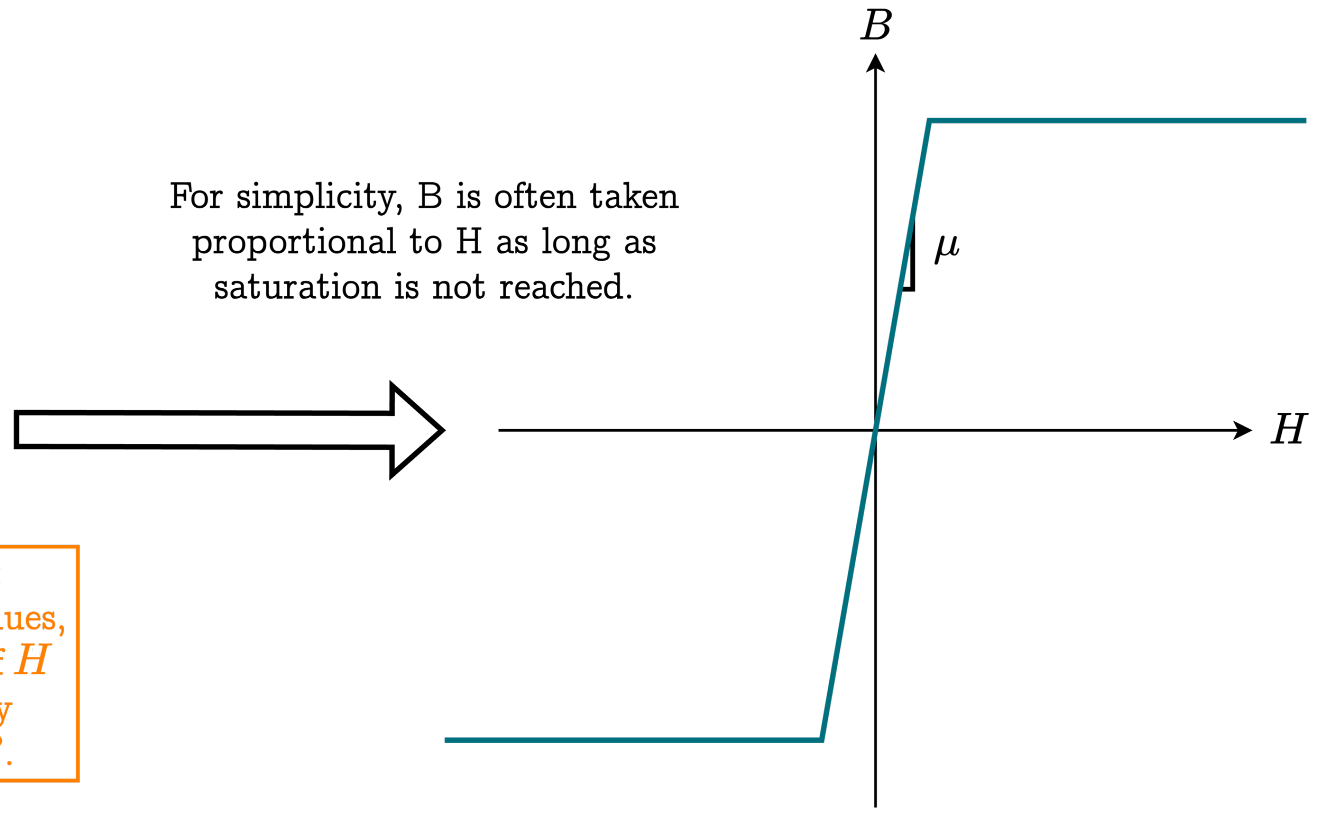
With $\vec{B}(t)$ uniform over S and considering the RMS values:

$$\phi = B\ S = \mu\ H\ S$$

Ferromagnetic materials

Ferromagnetic materials have a large magnetic permeability μ. For this reason, they are often used to handle high magnetic fluxes. They however exhibit hysteresis and saturation:





Reluctance and magnetic circuit

From Ampere's law (slide 5): $Hl = NI = \mathcal{F}$
From magnetic constitutive law (slide 6): $\phi = \mu HS$

$$\mathcal{F} = N\,I = \frac{l}{\mu\,S}\phi = \mathcal{R}\,\phi$$

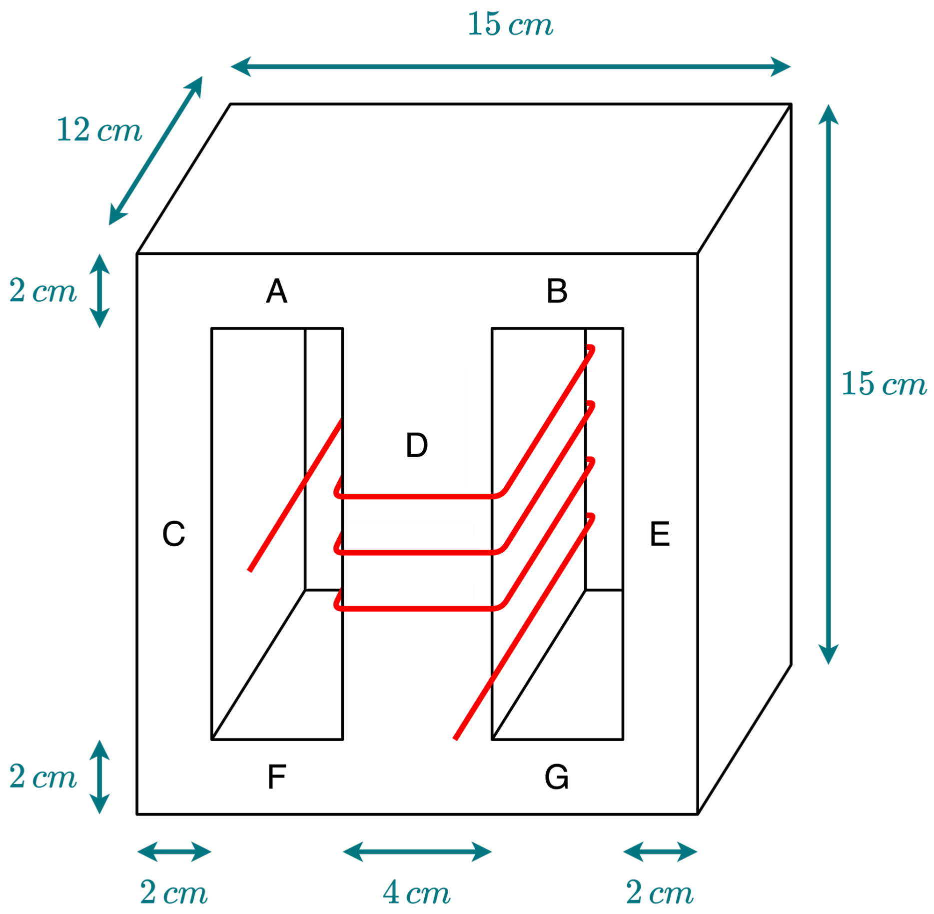
Magnetomotive force ($\mathcal{F}$) — Reluctance ($\mathcal{R}$) — Magnetic flux (ϕ)

The relation linking magnetomotive force, reluctance and magnetic flux is similar to the Ohm's law linking voltage, resistance and current:

Ohm's law: $V = R\,I$	Magnetic circuit equation: $\mathcal{F} = \mathcal{R}\,\phi$
Pouillet's law: $R = \frac{l}{\sigma\,S}$	Magnetic reluctance formula: $\mathcal{R} = \frac{l}{\mu\,S}$
σ is the conductivity $\left[\frac{S}{m}\right]$	μ is the permeability $\left[\frac{H}{m}\right]$

Exercise 6

Consider an inductor made of an iron core as depicted hereunder and a 60-turn winding, wounded around the central leg.



1. Draw an equivalent magnetic circuit of the inductor.

2. Compute the total reluctance of this circuit, considering a relative permeability μ_r of 1500 for the iron. Deduce the inductance from it.

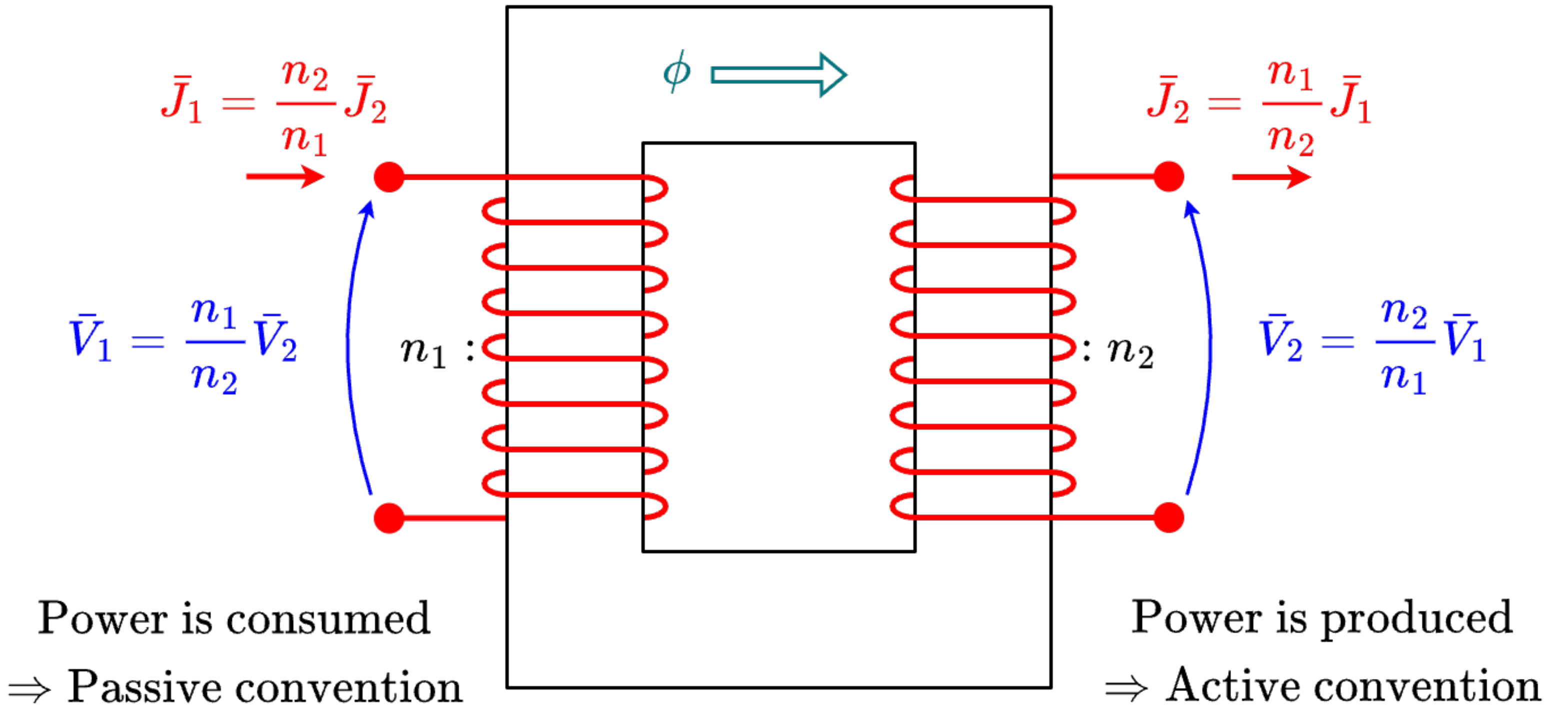
3. Do the same computation as in the previous steps, but now considering a constant air gap of 0.1 mm in each leg.

Transformers

The ideal transformer
The real transformer
Shifting impedances
Open-circuit and short-circuit tests
Exercise 7

The ideal transformer

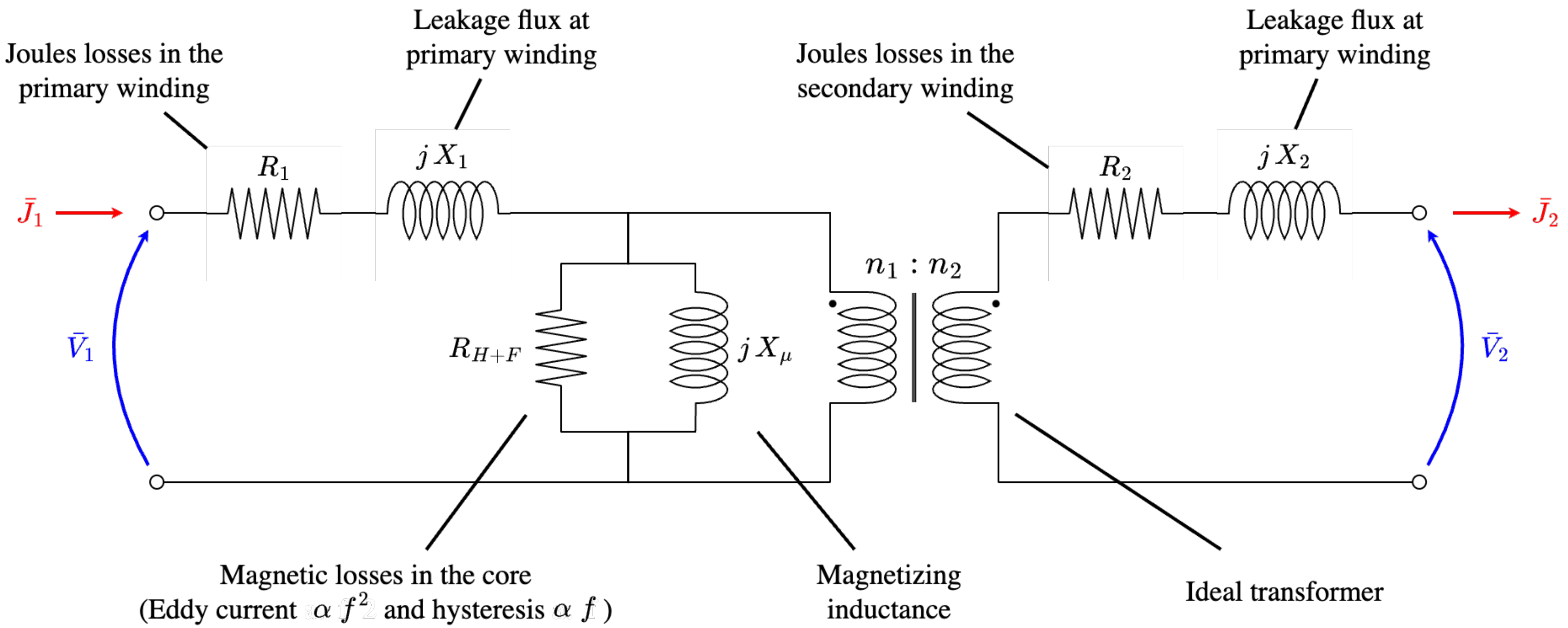
A transformer consists of two or more coils wrapped around a magnetic core, used to increase or decrease an AC voltage/current:



In an ideal transformer:

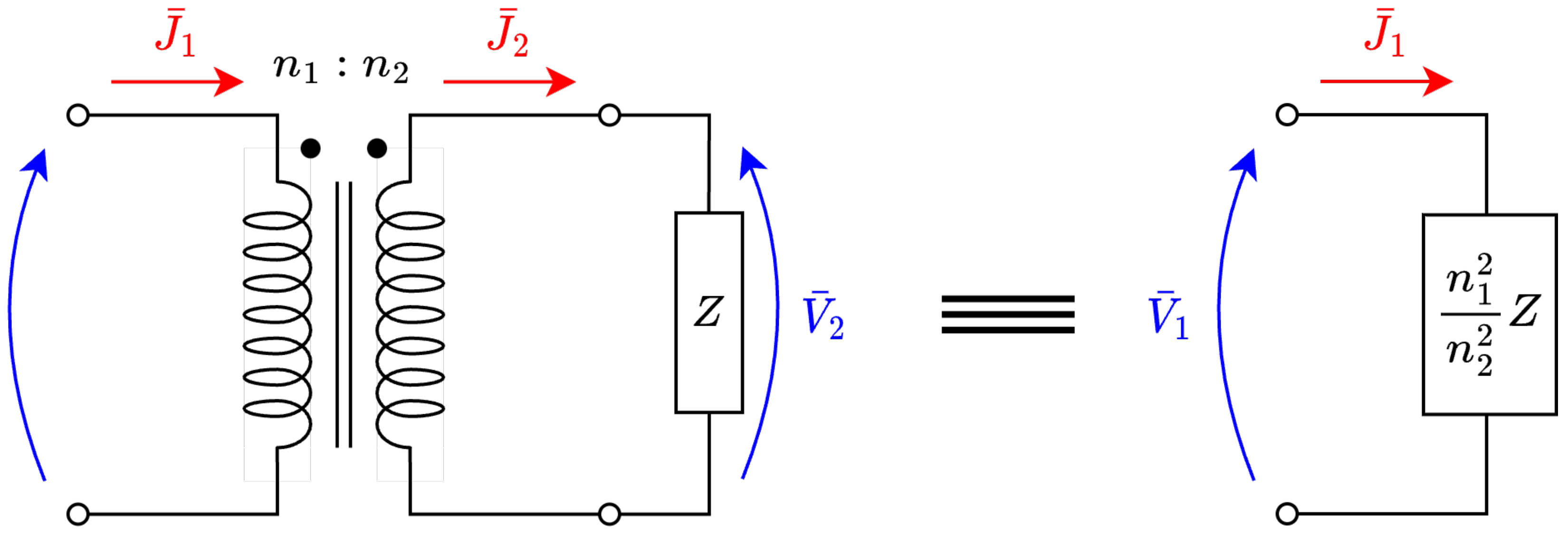
$$n_2\,\bar{V}_1 = n_1\,\bar{V}_2 \qquad \text{and} \qquad n_1\,\bar{J}_1 = n_2\,\bar{J}_2$$

The real transformer



In practice, transformers are built to minimize the losses ➔ $R_1, R_2, X_1, X_2 \ll R_{H+F},\ X_\mu$

Shifting impedances



$$n_2 \, \bar{V}_1 = n_1 \, \bar{V}_2$$

$$n_1 \, \bar{J}_1 = n_2 \, \bar{J}_2$$

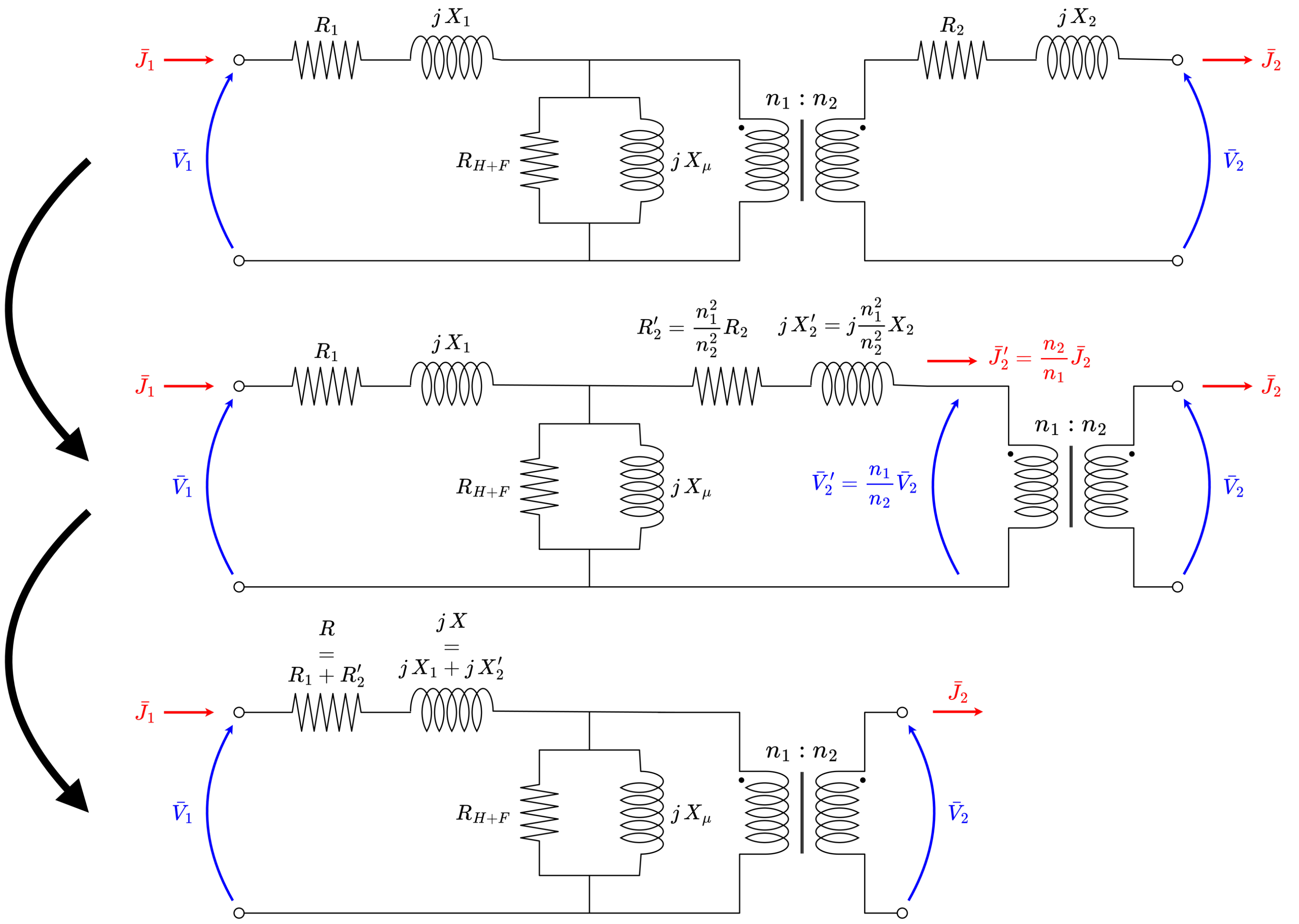
Seen from the secondary, the impedance is $Z = \dfrac{\bar{V}_2}{\bar{J}_2}$

Seen from the primary, the impedance is $Z' = \dfrac{\bar{V}_1}{\bar{J}_1} = \dfrac{\bar{V}_2\left(\frac{n_1}{n_2}\right)}{\bar{J}_2\left(\frac{n_2}{n_1}\right)} = \dfrac{n_1^2}{n_2^2}\,\dfrac{\bar{V}_2}{\bar{J}_2} = \dfrac{n_1^2}{n_2^2}\, Z$

The real transformer – impedances gathered at primary

One can shift the impedances from the secondary to primary side of the ideal transformer

R'_2 and $j\,X'_2$ can pass on the other side of the magnetizing branch since $R'_2, X'_2 \ll R_{H+F}, X_\mu$



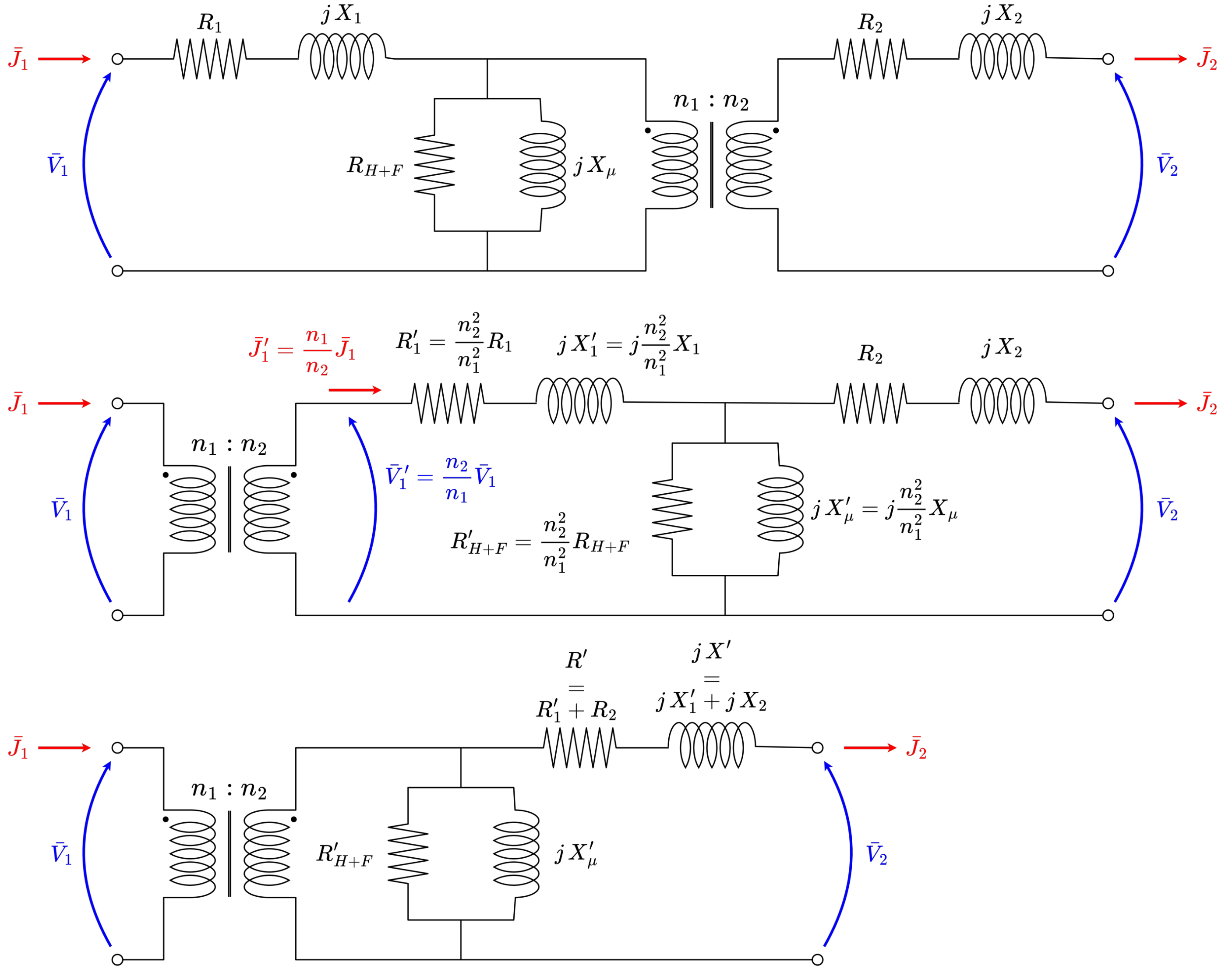
The real transformer – impedances gathered at secondary

One can shift the impedances from the primary to secondary side of the ideal transformer

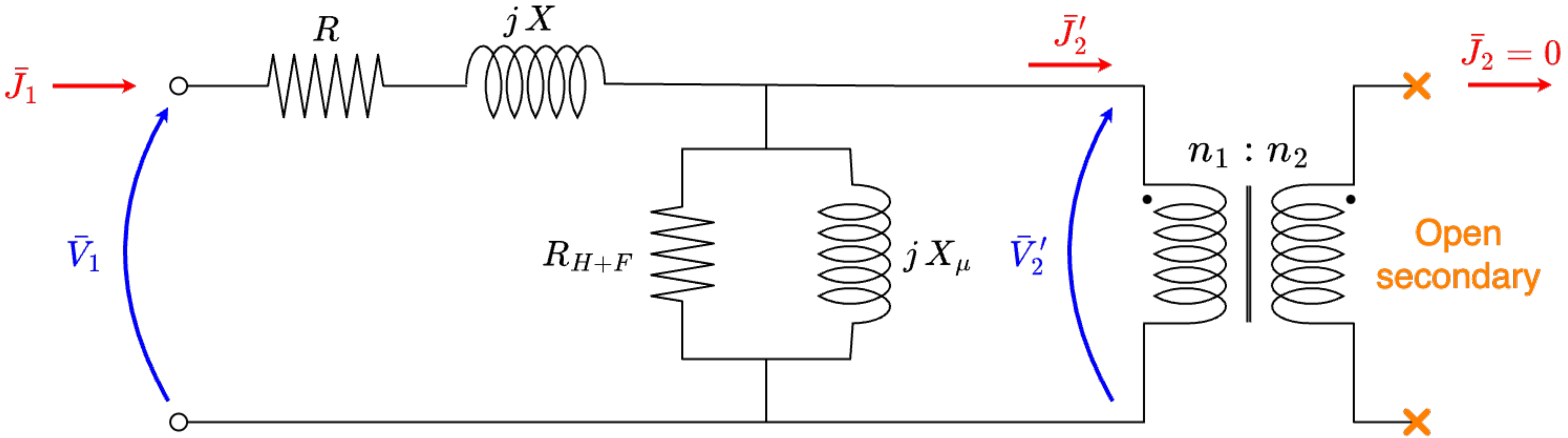
R_1' and $j\,X_1'$ can pass

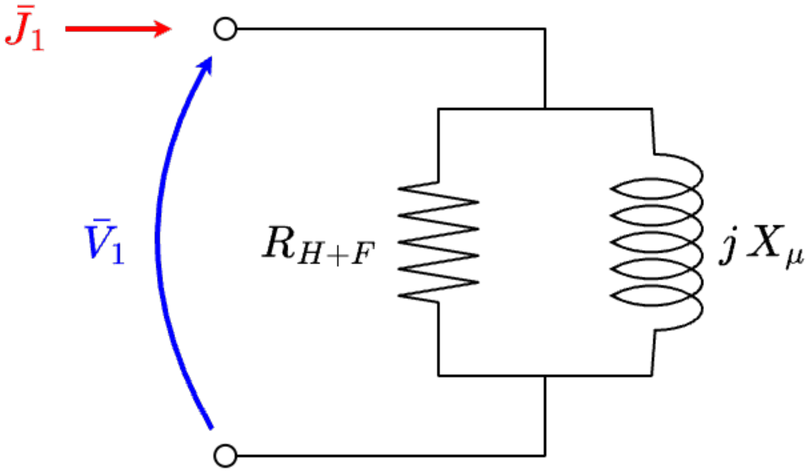
on the other side of the magnetizing branch since

$R_1', X_1' \ll R_{H+F}, X_\mu$



Open circuit test





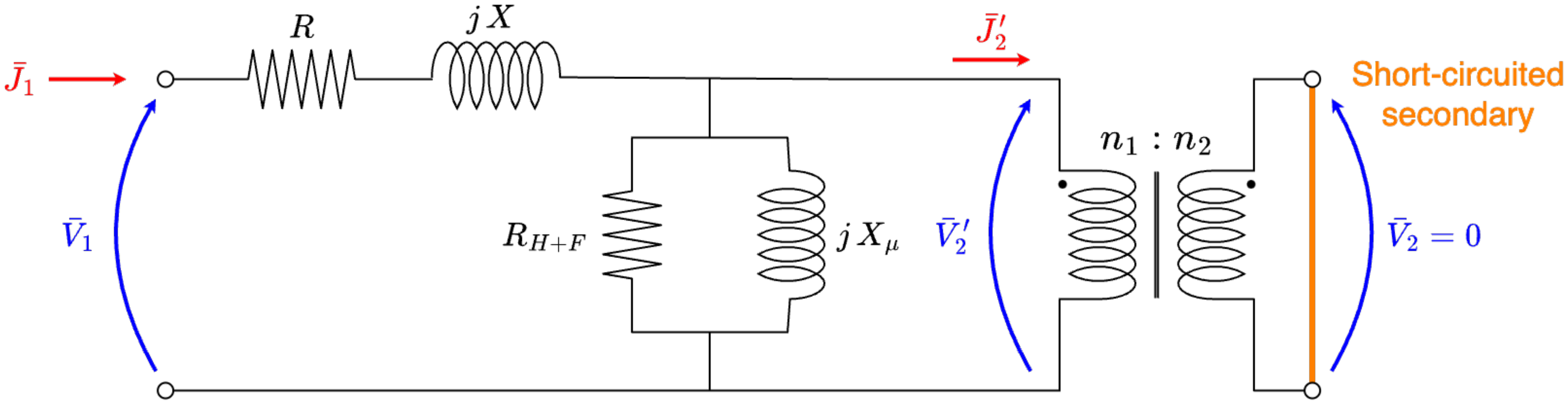
$\bar{J}_2 = 0 \;\rightarrow \bar{J}_2' = 0$

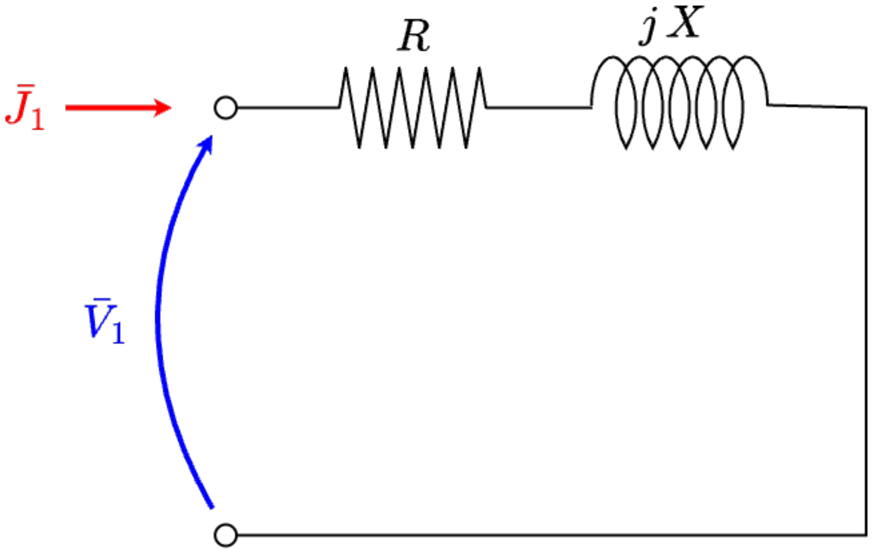
$R, X \ll R_{H+F}, X_\mu$
$\rightarrow \bar{V}_2' \gg (R + jX)\,\bar{J}_1$
$\rightarrow$ R and X can be neglected

$$P \approx \frac{V_1^2}{R_{\mathrm{H+}F}}$$

$$Q \approx \frac{V_1^2}{X_\mu}$$

Short-circuit test





$\bar{V}_2 = 0 \;\rightarrow \bar{V}_2' = 0$

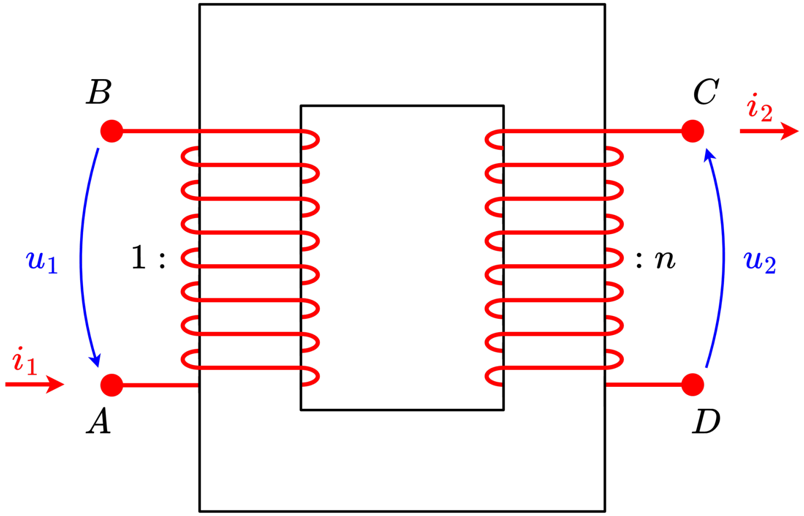
→ Voltage drop only for R and X

$$P \approx R\,J_1^2$$

$$Q \approx X\,J_1^2$$

Exercise 7

Two tests are performed on the transformer illustrated hereunder:



- Using open secondary winding, the transformer generates a voltage of RMS value $U_{2o} = 100\,V$ at the secondary winding, for an applied voltage of RMS value $U_{1o} = 20\,V$ with a drawn current intensity of RMS value $I_{1o} = 3.2\,A$ and a consumed power $P_{1o} = 8\,W$.

- Using short-circuited secondary winding, a voltage of RMS value $U_{1s} = 0.8\,V$ for a total power of $P = 24\,W$ is measured, causing a current flow of RMS value $I_{2s} = 10\,A$ through the secondary winding.

Considering a simplified equivalent model of the transformer (resistances and inductances gathered and moved to the secondary winding):

1. Calculate the transformer ratio n.

2. Calculate the resistance R'_{H+F} and the magnetizing inductance L'_{μ}, placed at the secondary of the transformer.

3. Compute the values of the resistance R' and the reactance X' corresponding to the Joule losses and the leakage reactance, placed at the secondary of the transformer.

Exercise 7

Using the transformer connected to a load on the secondary side drawing a current of RMS value $I_2 = 12\,A$ with a power factor $\cos(\varphi) = 0.8$ (the current is lagging the voltage), an RMS voltage $U_1 = 20\,V$ is applied to the primary winding.

4. Calculate the RMS voltage U_2 appearing across the secondary winding.
   (What wise approximation can be made here?)

5. Deduce the active power P_2 provided to the load.

6. Calculate the RMS current I_1 on the primary side.

7. Compute the transformer efficiency η.